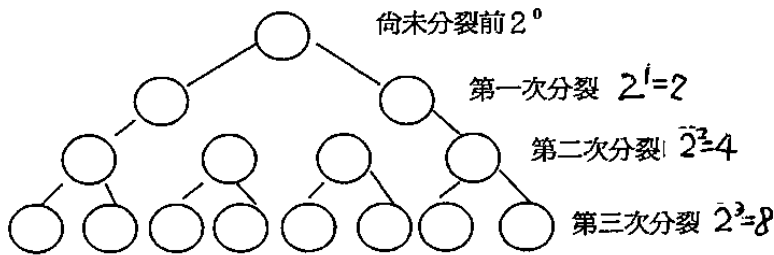


## 2.1 指數



細胞分裂時，一個變 2 個，兩個又變 4 個，分裂第三次成為 8 個，第四次 16 個，紀錄為

$$2^0=1 \quad \text{讀作 2 的 0 次方}$$

$$2^1=2 \quad \text{讀作 2 的 1 次方}$$

$$2^2=2 \times 2 = 4 \quad \text{讀作 2 的 2 次方}$$

$$2^3=2 \times 2 \times 2 = 8 \quad \text{讀作 2 的 3 次方}$$

$$2^4=2 \times 2 \times 2 \times 2 \quad \text{讀作 2 的 4 次方}$$

細胞尚未分裂前為  $2^0=1$ ，任何數的零次方=1，除了  $0^0$  無意義

討論：

$$3^0=1, \quad 3^1=3, \quad 3^2=9, \quad 3^3=27$$

$$5^0=1, \quad 5^1=5, \quad 5^2=25, \quad 5^3=125$$

$$10^0=1, \quad 10^1=10, \quad 10^2=100, \quad 10^3=1000$$

$$(-2)^0=1, \quad (-2)^1=-2, \quad (-2)^2=(-2) \cdot (-2)=4,$$

$$(-2)^3=(-2) \cdot (-2) \cdot (-2)=-16$$

$$\left(\frac{1}{2}\right)^0 = 1 \quad \left(\frac{1}{2}\right)^1 = \frac{1}{2}, \quad \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \quad \left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$(0.9)^0 = 1 \quad (0.9)^1 = 0.9 \quad (0.9)^2 = 0.81$$

練習：

$$2^0 = 1, \quad 2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16$$

$$2^5 = 32, \quad 2^6 = 64, \quad 2^7 = 128, \quad 2^8 = 256, \quad 2^9 = 512$$

$$99^0 = 1 \quad 1357^0 = 1$$

$$7^0 = 1 \quad 7^1 = 7 \quad 7^2 = 49 \quad (-7)^1 = -7 \quad (-7)^2 = 49$$

$$\left(\frac{1}{7}\right)^0 = 1 \quad \left(\frac{1}{7}\right)^2 = \frac{1}{49} \quad \left(\frac{1}{10}\right)^0 = 1 \quad \left(\frac{1}{10}\right)^1 = \frac{1}{10}$$

$$\left(\frac{1}{10}\right)^2 = \frac{1}{100} \quad \left(\frac{1}{10}\right)^3 = \frac{1}{1000} \quad \left(\frac{1}{10}\right)^4 = \frac{1}{10000}$$

負指數次方

$$x^{-n} = \frac{1}{x^n}$$

討論：

$$2^{-1} = \frac{1}{2}, \quad 2^{-2} = \frac{1}{2^2}, \quad 2^{-3} = \frac{1}{2^3}, \quad 2^{-4} = \frac{1}{2^4}$$

練習：

$$3^{-1} = \frac{1}{3} \quad 3^{-2} = \frac{1}{9} \quad 3^{-3} = \frac{1}{27} \quad 5^{-1} = \frac{1}{5} \quad 5^{-2} = \frac{1}{25}$$

$$5^{-3} = \frac{1}{125} \quad 10^{-1} = \frac{1}{10} \quad 10^{-2} = \frac{1}{100} \quad 10^{-3} = \frac{1}{1000} \quad 7^{-1} = \frac{1}{7}$$

$$(-3)^{-1} = \frac{1}{(-3)^1} \quad (-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9} \quad (-3)^{-3} = \frac{1}{-27}$$

$$x^{-1} = \frac{1}{x} \quad x^{-2} = \frac{1}{x^2} \quad x^{-3} = \frac{1}{x^3}$$

討論： $\frac{1^{-2}}{5} = \frac{1}{\left(\frac{1}{5}\right)^2} = \frac{1}{\frac{1}{25}} = 25$

練習：

$$\left(\frac{1}{6}\right)^{-2} = \frac{1}{\left(\frac{1}{6}\right)^2} = \frac{1}{\frac{1}{36}} = 36$$

$$\left(\frac{1}{10}\right)^{-3} = \frac{1}{\left(\frac{1}{10}\right)^3} = \frac{1}{\frac{1}{1000}} = 1000$$

$$\left(\frac{1}{8}\right)^{-2} = \frac{1}{\left(\frac{1}{8}\right)^2} = \frac{1}{\frac{1}{64}} = 64$$

指數的運算

一、指數相乘次方相加  $X^a \cdot X^b = X^{a+b}$

討論： $2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^5$

$$2^{10} \cdot 2^3 = 2^{10+3} = 2^{13}$$

$$3^5 \cdot 3^2 = 3^{5+2} = 3^7$$

討論： $X \cdot X = X^2$        $X \cdot X \cdot X = X^3$        $X^2 \cdot X^3 = X \cdot X \cdot X \cdot X \cdot X = X^5$

練習：

$$10^2 \cdot 10^3 = 10^5 \quad 10^{23} \cdot 10^{13} = 10^{36} \quad x^5 \cdot x^{13} = x^{18}$$

二、次方再次方則指數相乘  $(X^a)^b = X^{a \cdot b}$

$$(2^3)^2 = (2^3) \cdot (2^3) = 2^{3 \cdot 2} = 2^6$$

討論：

$$(2^3)^5 = 2^{15} \quad (5^{\frac{1}{2}})^2 = 5^1 \quad (5^{\frac{1}{3}})^6 = 5^{\frac{1}{3} \times 6} = 5^2$$

練習：

$$(5^{13})^2 = 5^{26}$$

$$(6^3)^4 = 6^{12}$$

$$(10^4)^{-3} = 10^{-12}$$

$$(6^{-3})^4 = 6^{-12}$$

$$(7^5)^{\frac{1}{5}} = 7^1$$

$$(6^{\frac{3}{2}})^4 = 6^6$$

$$\left(5^{-\frac{2}{5}}\right)^5 = 5^{-2} = \frac{1}{25}$$

$$(6^{\frac{1}{4}})^4 = 6^1$$

$$(125^{\frac{1}{3}})^3 = 125$$

$$(81^{\frac{1}{4}})^2 = 81^{\frac{1}{2}} = 9$$

根號~分數指數

$$1^{\frac{1}{2}} = \sqrt{1} \quad 1^2 = 1 \Leftrightarrow \sqrt{1} = 1^{\frac{1}{2}} = 1$$

$$2^{\frac{1}{2}} = \sqrt{2} \quad 2^2 = 4 \Leftrightarrow \sqrt{4} = 4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2$$

$$3^{\frac{1}{2}} = \sqrt{3} \quad 3^2 = 9 \Leftrightarrow \sqrt{9} = 9^{\frac{1}{2}} = (3^3)^{\frac{1}{2}} = 3$$

$$4^{\frac{1}{2}} = \sqrt{4} \quad 4^2 = 16 \Leftrightarrow \sqrt{16} = 16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} = 4$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

討論

$$\sqrt{100} = 10$$

$$\sqrt{81} = 9$$

$$\sqrt{49} = 7$$

$$\sqrt{36} = 6$$

$$\sqrt{25} = 5$$

$$\sqrt[3]{27} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3$$

練習

- |                           |  |  |
|---------------------------|--|--|
| 1. $2^0 = 1$              | 19. $4^2 = 16$                                       | 38. $(-2)^2 = 4$                       |
| 2. $2^1 = 2$              | 20. $5^2 = 25$                                       | 39. $(-2)^3 = -8$                      |
| 3. $2^2 = 4$              | 22. $7^2 = 49$                                       | 40. $(-2)^4 = 16$                      |
| 4. $2^3 = 8$              | 23. $8^2 = 64$                                       | 41. $(-2)^5 = -32$                     |
| 5. $2^4 = 16$             | 24. $9^2 = 81$                                       | 42. $(-1)^0 = 1$                       |
| 6. $2^5 = 32$             | 25. $10^2 = 100$                                     | 43. $(-1)^1 = -1$                      |
| 7. $2^6 = 64$             | 26. $11^2 = 121$                                     | 44. $(-1)^2 = 1$                       |
| 8. $2^7 = 128$            | 27. $12^2 = 144$                                     | 45. $(-1)^3 = -1$                      |
| 9. $2^8 = 256$            | 28. $13^2 = 169$                                     | 46. $(-1)^4 = 1$                       |
| 10. $2^9 = 512$           | 29. $14^2 = 196$                                     | 47. $(\frac{1}{3})^0 = 1$              |
| 11. $2^{10} = 1024$       | 30. $10^3 = 1000$                                    | 48. $(\frac{1}{3})^1 = \frac{1}{3}$    |
| 12. $3^0 = 1$             | 31. $10^4 = 10000$                                   | 49. $(\frac{1}{3})^2 = \frac{1}{9}$    |
| 13. $3^1 = 3$             | 32. $10^5 = 100000$                                  | 50. $(\frac{2}{3})^3 = \frac{8}{27}$   |
| 14. $3^2 = 9$             | 33. $10^6 = 1000000$                                 | 51. $(-\frac{1}{2})^0 = 1$             |
| 15. $3^3 = 27$            | 34. $(\frac{1}{2})^2 = \frac{1}{4}$                  | 52. $(-\frac{1}{5})^1 = -\frac{1}{5}$  |
| 16. $3^4 = 81$            | 35. $(\frac{1}{2})^3 = \frac{1}{8}$                  | 53. $(\frac{1}{5})^2 = \frac{1}{25}$   |
| 17. $4^0 = 1$             | 36. $(\frac{1}{2})^4 = \frac{1}{16}$                 | 54. $(-\frac{1}{5})^3 = \frac{1}{125}$ |
| 18. $4^1 = 4$             | 37. $(-2)^1 = -2$                                    | 55. $(\frac{1}{5})^4 = \frac{1}{625}$  |
| 1. $5^{-1} = \frac{1}{5}$ | 21. $(3X)^{-3} = \frac{1}{(3X)^3} = \frac{1}{27X^3}$ | 41. $16^{\frac{1}{2}} = \sqrt{16} = 4$ |

2.  $5^{-2} = \frac{1}{25}$
3.  $5^{-3} = \frac{1}{125}$
4.  $(-5)^{-1} = \frac{1}{-5}$
5.  $(-5)^{-2} = \frac{1}{25}$
6.  $(\frac{3}{5})^{-1} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$
7.  $(\frac{3}{5})^{-2} = \frac{1}{(\frac{3}{5})^2} = \frac{1}{\frac{9}{25}} = \frac{25}{9}$
8.  $(-\frac{1}{5})^{-1} = \frac{1}{(-\frac{1}{5})} = -5$
9.  $(-\frac{1}{5})^{-2} = 25$
10.  $5^{-2} \cdot 5^{-3} =$
11.  $(5^2)^3 = 5^6$
12.  $(5^{-2})^3 = 5^{-6}$
13.  $(5^2)^{-3} = 5^{-6}$
14.  $(\frac{2}{5})^{-3} = \frac{1}{\frac{8}{125}} = \frac{125}{8}$
15.  $(-\frac{2}{5})^{-3} = \frac{1}{(-\frac{2}{5})^3} = \frac{1}{-\frac{8}{125}} = -\frac{125}{8}$
16.  $(\frac{9}{16})^{-1} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$
17.  $X^3 \cdot X^{-4} = X^{-2} = \frac{1}{X^2}$
18.  $X^3 \cdot X^4 = X^7$
19.  $(X^3)^4 = X^{12}$
20.  $(3X)^{-3} = \frac{1}{(3X)^3} = \frac{1}{27X^3}$
22.  $(3X)^{-1} = \frac{1}{3X}$
23.  $1^2 = 1$
24.  $2^2 = 4$
25.  $3^2 = 9$
26.  $4^2 = 16$
27.  $5^2 = 25$
28.  $6^2 = 36$
29.  $7^2 = 49$
30.  $8^2 = 64$
31.  $9^2 = 81$
32.  $10^2 = 100$
33.  $11^2 = 121$
34.  $4^{\frac{1}{2}} = 2$
35.  $9^{\frac{1}{2}} = 3$
36.  $16^{\frac{1}{2}} = 4$
37.  $8^{\frac{1}{3}} = 2$
38.  $16^{\frac{1}{4}} = 2$
39.  $27^{\frac{1}{3}} = 3$
40.  $(-8)^{\frac{1}{3}} = [(-2)^3]^{\frac{1}{3}} = -2$
42.  $16^{\frac{3}{2}} = (2^4)^{\frac{3}{2}} = 2^6$
43.  $16^{-\frac{3}{2}} = (2^4)^{-\frac{3}{2}} = 2^{-6}$
44.  $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3$
45.  $27^{-\frac{1}{3}} = (3^3)^{-\frac{1}{3}} = 3^{-1} = \frac{1}{3}$
46.  $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$
47.  $27^{-\frac{2}{3}} = (3^3)^{-\frac{2}{3}} = 3^{-2} = \frac{1}{9}$
48.  $(-27)^{\frac{2}{3}} = (-3)^{3 \cdot \frac{2}{3}} = (-3)^2 = 9$
49.  $(-27)^{-\frac{2}{3}} = (-3)^{3 \cdot (-\frac{2}{3})} = (-3)^{-2} = \frac{1}{9}$
50.  $\sqrt{1} = 1 = \frac{1}{1}$
51.  $\sqrt{4} = 2$
52.  $\sqrt{9} = 3$
53.  $\sqrt{16} = 4$
54.  $\sqrt{25} = 5$
55.  $\sqrt{36} = 6$
56.  $\sqrt{49} = 7$
57.  $\sqrt{64} = 8$
58.  $\sqrt{81} = 9$
59.  $\sqrt{100} = 10$
60.  $\sqrt{121} = 11$