

## 2.3 對數函數

$$2^3 = 8 \Leftrightarrow \log_2 8 = 3 = \log_2 2^3 \quad \text{讀作以 2 為底 8 的對數}=3$$

討論：

$$(1) \log_2 2 = 1$$

$$(2) \log_2 4 = \log_2 2^2 = 2$$

$$(3) \log_2 16 = \log_2 2^4 = 4$$

$$(4) \log_2 1 = \log_2 2^0 = 0$$

$$(5) \log_2 \frac{1}{4} = \log_2 2^{-2} = -2$$

$$(6) \log_{10} 10 = 1$$

$$(7) \log_{10} 100 = \log_{10} 10^2 = 2$$

$$(8) \log_{10} \frac{1}{10} = \log_{10} 10^{-1} = -1$$

$$(9) \log_{10} \frac{1}{100} = \log_{10} \frac{1}{10^2} = \log_{10} 10^{-2} = -2$$

$$(10) \log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$$

$$(11) \log_3 1 = \log_3 3^0 = 0$$

$$(12) \log_3 3 = 1$$

$$(13) \log_3 9 = \log_3 3^2 = 2$$

$$(14) \log_3 27 = \log_3 3^3 = 3$$

$$(15) \log_3 \frac{1}{3} = \log_3 3^{-1} = -1$$

$$(16) \log_{10} 1 = \log_{10} 10^0 = 0$$

$$(17) \log_5 25 = \log_5 5^2 = 2$$

$$(18) \log_5 125 = \log_5 5^3 = 3$$

$$(19) \log_5 \frac{1}{5} = \log_5 5^{-1} = -1$$

$$(20) \log_5 \sqrt{5} = \log_5 5^{\frac{1}{2}} = \frac{1}{2}$$

$$(21) \log_{10} 0.1 = \log_{10} \frac{1}{10} = \log_{10} 10^{-1} = -1$$

$$(22) \log_{10} 0.01 = \log_{10} 10^{-2} = -2$$

$$(23) \log_{10} \sqrt{10} = \log_{10} 10^{\frac{1}{2}} = \frac{1}{2}$$

$$(24) \log_7 \sqrt{7} = \log_7 7^{\frac{1}{2}} = \frac{1}{2}$$

### 指數與對數的練習

$$1. 3^0 = 1$$

$$21. \log_3 81 = 4$$

$$41. 5^{\frac{1}{2}} = \sqrt{5}$$

$$2. 3^1 = 3$$

$$22. \log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = -2$$

$$42. 5^{\frac{1}{3}} = \sqrt[3]{5}$$

$$3. 3^2 = 9$$

$$23. \log_3 \frac{1}{27} = \log_3 3^{-3} = -3$$

$$43. 5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}} = \frac{1}{\sqrt{5}}$$

$$4. 3^3 = 27$$

$$24. \log_{(-3)}(-27) = \log_{(-3)} (-3)^3 = 3$$

$$44. 5^{\frac{1}{3}} = \sqrt[3]{5}$$

$$5. 3^4 = 81$$

$$25. \log_{-3} 9 = \log_{(-3)} (-3)^2 = 2$$

$$45. (-5)^2 = 25$$

$$6. 3^{-1} = \frac{1}{3}$$

$$26. \log_{\frac{1}{3}} \frac{1}{9} = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^2 = 2$$

$$46. (-5)^3 = -125$$

$$7. 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$27. \log_{\frac{1}{3}} 9 = \log_{\frac{1}{3}} 3^2 = \log_{\frac{1}{3}} (3^{-1})^2 = -2$$

$$47. (-5)^{-1} = \frac{1}{-5}$$

$$8. 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$28. \log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = \frac{1}{2}$$

$$48. (-5)^2 = 25$$

$$9. (-3)^1 = -3$$

$$29. \log_{-3} \left(-\frac{1}{27}\right) =$$

$$49. (-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$$

$$= \log_{-3} \left(-\frac{1}{3}\right)^3 = \log_{-3} (-3)^{-3} = -3$$