

## 2.5 常用對數

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$$

$$\log_{10} 0.001 = \log_{10} 10^{-3} = -3$$

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$\log_{10} 0.01 = \log_{10} 10^{-2} = -2$$

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

$$\log_{10} 0.1 = \log_{10} 10^{-1} = -1$$

$$10^0 = 1$$

$$\log_{10} 1 = 0$$

$$10^1 = 10$$

$$\log_{10} 10 = 1$$

$$10^2 = 100$$

$$\log_{10} 100 = \log_{10} 10^2 = 2$$

$$10^3 = 1000$$

$$\log_{10} 1000 = \log_{10} 10^3 = 3$$

練習：

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$$1. 10^5 = 100000$$

$$1. \log_{10} 10000 = 4$$

$$2. 10^6 = 1000000$$

$$2. \log_{10} 100000 = 5$$

$$3. 10^{-7} = 0.0000001$$

$$3. \log_{10} 1000000 = 6$$

$$4. 10^{-6} = 0.000001$$

$$4. \log_{10} 0.0001 = -4$$

$$5. 10^{-5} = 0.00001$$

$$5. \log_{10} 0.001 = -3$$

$$6. 10^{-4} = 0.0001$$

$$6. \log_{10} 0.00001 = -5$$

以 10 為底的對數函數稱為常用對數函數，常用對數函數將

$10^{-n} \sim 10^n$  的數字對應到  $[-n, n]$

例如：

$$\log_{10} 10000 < \log_{10} 12345 < \log_{10} 100000$$

$$\rightarrow 4 < \log_{10} 12345 < 5 \quad \text{所以 } \log_{10} 12345 \cong 4.1$$

例如：

$\log_{10} x = 4.6$   $\log_{10} 10000 < \log_{10} x < \log_{10} 100000$  則  $x$  為 5 位數

$\log_{10} x = 5.3$  則  $x$  為 6 位數  $\because \log 100000 < \log x < \log 1000000$

$\log_{10} x = 6.8$  則  $x$  為 7 位數

$\log_{10} x = 4.6$  則  $x$  為 5 位數

練習

1.  $\log_{10} x = 2.3$  則  $x$  為 (3) 位數
2.  $\log_{10} x = 3.8$  則  $x$  為 (4) 位數
3.  $\log_{10} x = 7.6$  則  $x$  為 (8) 位數

對數的性質

1.  $\log_a 1 = 0$

例： $\log_{10} 1 = 0$

2.  $\log_a b = \frac{\log_{10} b}{\log_{10} a}$

$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2}$

3.  $\log_{10}(a \times b) = \log_{10} a + \log_{10} b$   $\log_{10} 6 = \log_{10} 2 + \log_{10} 3$

4.  $\log_{10} a^n = n \log_{10} a$   $\log_{10} 2^3 = 3 \log_{10} 2$

以 10 為底的對數稱為常用對數，以  $\log x$  表示  $\log_{10} x$

討論：

$$\log 2 = 0.3010, \log 3 = 0.4771, \log 5 = 0.6990$$

$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2} = \frac{0.6990}{0.3010} = 2.322$$

$$\log_{10} 6 = \log_{10} 2 + \log_{10} 3 = 0.3010 + 0.4771 = 0.7781$$

$$\begin{aligned}\log_{10} 2^3 &= \log(2 \times 2 \times 2) = \log 2 + \log 2 + \log 2 = 3 \log_{10} 2 \\ &= 3 \times 0.3010 = 0.9030\end{aligned}$$

$$\log 32 = \log 2^5 = 5 \log 2 = 5 \times 0.3010 = 1.5050$$

練習：

$$1. \log_3 5 = \frac{\log 5}{\log 3} = \frac{0.6990}{0.4771} = 1.4651$$

$$2. \log_3 6 = \frac{\log 6}{\log 3} = \frac{0.7781}{0.4771} = 1.6308$$

$$3. \log_5 2 = \frac{\log 2}{\log 5} = \frac{0.3010}{0.6990} = 0.4306$$

$$4. \log_6 5 = \frac{\log 5}{\log 6} = \frac{0.6990}{0.7781} = 0.8983$$

$$5. \log 16 = \log 2^4 = 4 \times \log 2 = 4 \times 0.3010 = 1.204$$

$$6. \log 15 = \log(3 \times 5) = \log 3 + \log 5 = 0.4771 + 0.6990 = 1.1761$$

$$7. \log 12 = \log(3 \times 2^2) = \log 3 + \log 2^2 = 0.4771 + 2 \times 0.3010 = 1.0791$$

$$8. \log 18 = \log(3 \times 6) = \log 3 + \log 6 = 0.4771 + 0.7781 = 1.2552$$

$$\begin{aligned}9. \log 60 &= \log(10 \times 2 \times 3) = \log 10 + \log 2 + \log 3 = 1 + 0.3010 + 0.4771 \\ &= 1.7781\end{aligned}$$

$$10. \log 9 = \log 3^2 = 2 \times \log 3 = 2 \times 0.4771 = 0.9542$$

$$11. \log 27 = \log 3^3 = 3 \times \log 3 = 3 \times 0.4771 = 1.4313$$

$$12. \log 81 = \log 3^4 = 4 \times \log 3 = 4 \times 0.4771 = 1.9084$$

$$13. \log 25 = \log 5^2 = 2 \times \log 5 = 2 \times 0.6990 = 1.398$$

使用常用對數表求對數

例如： $\log 1.35 = 0.1303$

log	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279

練習：

$$\log 1.07 = 0.0294$$

$$\log 1.23 = 0.0899$$

$$\log 1.48 = 0.1703$$

$$\log 1.54 = 0.1875$$

$$\log 1.6 = 0.2041$$

常用對數表可以查詢 1.00~9.99 的對數，

$$1 \leq x < 10, \quad \text{則 } \log 1 \leq \log x < \log 10,$$

所以  $0 \leq \log x < 1$ ，對數表的值介於 0 與 1 之間。

如果  $x > 10$  則須將  $x$  化為科學記號  $a \times 10^n$ ,  $1 \leq a < 10$ , 再用對數的性質及對數表來求對數。

討論：將下列各數化為科學記號

$$1350 = 1.350 \times 1000 = 1.350 \times 10^3$$

$$324 = 3.24 \times 100 = 3.24 \times 10^2$$

$$53.2 = 5.32 \times 10 = 5.32 \times 10^1$$

$$6391 = 6.391 \times 1000 = 6.391 \times 10^3$$

$$976000 = 9.76 \times 100000 = 9.76 \times 10^5$$

$$21500 = 2.15 \times 10000 = 2.15 \times 10^4$$

討論：

求  $\log 1350$

$$\begin{aligned} 1350 &= 1.350 \times 10^3 \\ \log 1350 &= \log(1.35 \times 10^3) \\ &= \log 1.350 + \log 10^3 \\ &= 0.1303 + 3 \\ &= 3.1303 \end{aligned}$$

練習：

求  $\log 324$

$$\begin{aligned} &= \log(3.24 \times 100) \\ &= \log 3.24 + \log 100 \\ &= 0.5105 + 2 \\ &= 2.5105 \end{aligned}$$

練習：

求  $\log 693$

$$\begin{aligned} &= \log(6.93 \times 100) \\ &= \log 6.93 + \log 10^2 \\ &= 0.8407 + 2 \\ &= 2.8407 \end{aligned}$$

練習：

求  $\log 976000$

$$\begin{aligned} &= \log(9.76 \times 10^5) \\ &= \log 9.76 + \log 10^5 \\ &= 0.9877 + 5 \\ &= 5.9877 \end{aligned}$$

練習：

求  $\log 69300$

$$\begin{aligned} &= \log(6.93 \times 10^4) \\ &= \log 6.93 + \log 10^4 \\ &= 0.8407 + 4 \\ &= 4.8407 \end{aligned}$$

練習：

求  $\log 532$

$$\begin{aligned} &= \log(5.32 \times 100) \\ &= \log 5.32 + \log 10^2 \\ &= 0.7259 + 2 \\ &= 2.7259 \end{aligned}$$

練習：

求  $\log 21500$

$$\begin{aligned} &= \log(2.15 \times 10^4) \\ &= \log 2.15 + \log 10^4 \\ &= 0.3324 + 4 \\ &= 4.3324 \end{aligned}$$

討論：將下列各數化為科學記號

$$0.1350 = 1.350 \times 0.1 = 1.350 \times 10^{-1}$$

$$0.0135 = 1.35 \times 0.01 = 1.35 \times 10^{-2}$$

$$0.00135 = 1.35 \times 0.001 = 1.35 \times 10^{-3}$$

$$0.00324 = 3.24 \times 0.001 = 3.24 \times 10^{-3}$$

$$0.6391 = 6.391 \times 10^{-1}$$

$$0.00976 = 9.76 \times 0.001 = 9.76 \times 10^{-3}$$

$$0.532 = 5.32 \times 0.1 = 5.32 \times 10^{-1}$$

$$0.0215 = 2.15 \times 0.01 = 2.15 \times 10^{-2}$$

討論：

求  $\log 0.532$

$$0.532 = 5.32 \times 10^{-1}$$

$$\log 0.532 = \log(5.32 \times 10^{-1})$$

$$= \log 5.32 + \log 10^{-1}$$

$$= 0.7259 + (-1)$$

$$= -0.2741$$

練習：

求  $\log 0.0215$

$$= \log(2.15 \times 10^{-2})$$

$$= \log 2.15 + \log 10^{-2}$$

$$= 0.3324 + (-2)$$

$$= -1.6676$$

練習：

求  $\log 0.00976$

$$= \log(9.76 \times 0.001)$$

$$= \log 9.76 + \log 0.001$$

$$= 0.9894 + (-3)$$

$$= -2.0106$$

練習：

求  $\log 0.639$

$$= \log(6.39 \times 0.1)$$

$$= \log 6.39 + \log 0.1$$

$$= 0.8055 + (-1)$$

$$= -0.1945$$

練習：

求  $\log 0.00324$

$$= \log(3.24 \times 0.001)$$

$$= \log 3.24 + \log 0.001$$

$$= 0.5105 + (-3)$$

$$= -2.4895$$