

數學(四)三角函數多項式運算練習

1. 一圈  $360^\circ = 2\pi$  徑

2. 將下列角度化為弧度

$90^\circ = (\frac{\pi}{2})$  徑

$30^\circ = (\frac{\pi}{6})$  徑

$60^\circ = (\frac{\pi}{3})$  徑

$120^\circ = (\frac{2\pi}{3})$  徑

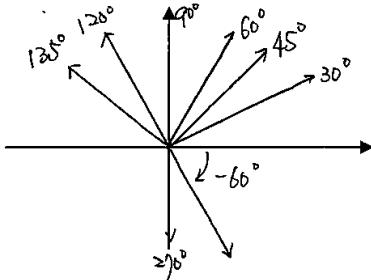
$45^\circ = (\frac{\pi}{4})$  徑

$-60^\circ = (-\frac{\pi}{3})$  徑

$270^\circ = (\frac{3}{2}\pi)$  徑

$135^\circ = (\frac{3}{4}\pi)$  徑

在座標系上畫出上列各角度



3. 將下列角度化為弧度

$\frac{\pi}{3}$  徑 = (60) 度

$-\frac{\pi}{2}$  徑 = (-90) 度

$\frac{\pi}{6}$  徑 = (30) 度

$\frac{\pi}{4}$  徑 = (45) 度

$\frac{5\pi}{6}$  徑 = (150) 度

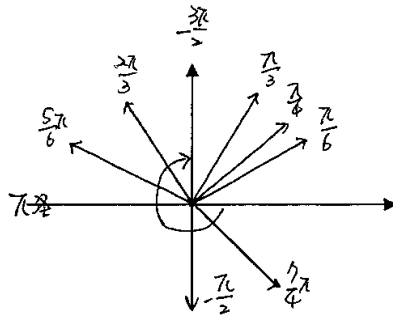
$\frac{2\pi}{3}$  徑 = (120) 度

$\pi$  徑 = (180) 度

$\frac{7\pi}{4}$  徑 = (315) 度

$-\frac{3\pi}{2}$  徑 = (-270) 度

在座標系上畫出上列各角度



4. 直角三角形斜邊長 1,

角度	$30^\circ$	$45^\circ$	$60^\circ$
角度對邊長	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

5. 計算下列各三角函數

$\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\cot 30^\circ = \sqrt{3}$

$\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\csc 30^\circ = 2$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$

$\cot 45^\circ = 1$

$\sec 45^\circ = \sqrt{2}$

$\csc 45^\circ = \sqrt{2}$

$\sin 120^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = -\frac{1}{2}$

$\tan 120^\circ = -\sqrt{3}$

$\cot 120^\circ = \frac{1}{\sqrt{3}}$

$\sec 120^\circ = -2$

$\csc 120^\circ = \frac{2}{\sqrt{3}}$

$\sin 0^\circ = 0$

$\sin 90^\circ = 1$

$\sin 180^\circ = 0$

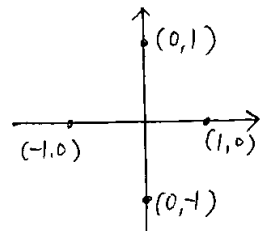
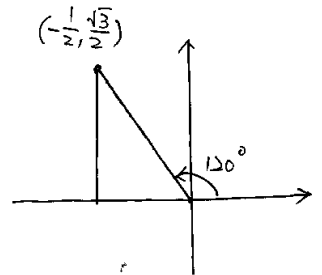
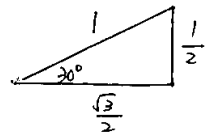
$\sin 360^\circ = 0$

$\cos 0^\circ = 1$

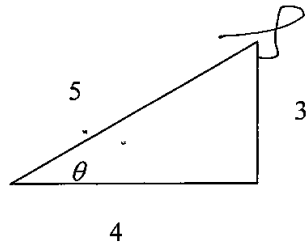
$\cos 90^\circ = 0$

$\cos 180^\circ = -1$

$\cos 360^\circ = 1$



10. 求下列角度的六個三角函數



6. 寫出三角函數的性質

(1) 倒數關係

$$\cot \theta = \frac{1}{\tan \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

(2) 平方關係

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

(3) 商數關係

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\begin{aligned} \sin \theta &= \frac{3}{5} \\ \cos \theta &= \frac{4}{5} \\ \tan \theta &= \frac{3}{4} \\ \cot \theta &= \frac{4}{3} \\ \sec \theta &= \frac{5}{4} \\ \csc \theta &= \frac{5}{3} \end{aligned}$$

7.  $\sin^2 10^\circ + \cos^2 10^\circ = 1$

$\sin^2 30^\circ + \cos^2 30^\circ = 1$

$\sec^2 10^\circ - \tan^2 10^\circ = 1$

多項式運算

$p(x) = x^3 + 4x^2 + 2x - 3, q(x) = x + 2$

求

$p(x) + q(x) = x^3 + 4x^2 + 3x - 1$

$p(x) - q(x) = x^3 + 4x^2 + x - 5$

$p(x) \cdot q(x) = x^4 + 6x^3 + 10x^2 + x - 6$

$p(x) \div q(x) = (x^2 + 2x - 2) \text{ 餘 } 1$

8. 寫出和角公式

$\cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

$\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$\sin(x - y) = \sin x \cdot \cos y - \cos x \cdot \sin y$

將下列各式因式分解

$x^2 - 36 = (x - 6)(x + 6)$

$x^2 - 16 = (x - 4)(x + 4)$

$x^2 - 25 = (x - 5)(x + 5)$

$y^2 - 1 = (y - 1)(y + 1)$

$x^2 - 5x - 6 = (x - 2)(x - 3)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$x^2 - 3x + 2 = (x - 1)(x - 2)$

$x^2 - 6x + 8 = (x - 2)(x - 4)$

$2x^2 - 3x - 2 = (2x + 1)(x - 2)$

$2x^2 - x - 10 = (2x - 5)(x - 2)$

9. 利用和角公式求

$\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

$\cos 105^\circ = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

$\sin 15^\circ = \sin(60^\circ - 45^\circ)$

$= \sin 60^\circ \cdot \cos 45^\circ - \cos 60^\circ \cdot \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

$\cos 15^\circ = \cos(60^\circ - 45^\circ)$

$= \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$

利用餘式定理求  $p(x) \div q(x)$  的餘數

$p(x) = x^3 + 4x^2 + 2x - 3, q(x) = x - 1$  餘 4

$p(x) = x^3 + 2x - 3, q(x) = x + 2$  餘 -5

$p(x) = 4x^2 + 2x - 3, q(x) = x - 3$  餘 39